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超弹性 N 形杆纯弯曲力矩理论建模和试验验证

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摘要: 为了使 N 形杆在空间结构中顺利展开, 对其缠绕过程中的力学特性进行了分析。基于协变基向量法建立了纯弯曲条件下各向同性单片带簧在缠绕时应变能理论模型, 并采用最小势能原理推导了弯矩的理论模型, 然后通过拟合曲率推导了 N 形杆缠绕时的理论模型。研制了 4 根 N 形杆样件, 并搭建了试验平台, 使用数显式推拉力计分别对杆件在缠绕过程的拉力进行了 5 次测试, 然后求出峰值力矩, 发现试验值与理论值的偏差不大于 8.83%, 偏差平均值为 2.813%, 从而验证了理论模型的准确性。该研究可用来分析超弹性杆在纯弯曲状态下的峰值力矩, 对超弹性杆在宇航空间中的应用具有重大意义。

关键词: 可折展机构; N 形杆; 协变基向量法; 缠绕弯曲; 理论建模

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Theoretical Modeling of Pure Bending Moment of Super-Elastic N-Shaped Boom and Its Experimental Verification

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Abstract: In order to make the N-shaped boom develop smoothly in the space structure, the mechanical properties of its winding process are analyzed. Based on the covariant basis vector method, a theoretical model of the strain energy of an isotropic single tape spring during winding under pure bending conditions is established, and the theoretical model of the bending moment is derived by using the principle of minimum potential energy, and then the bending moment of the N-shaped boom is derived by fitting the curvature. Four N-shaped boom samples are developed and an experimental platform is built. The tensile force of the four N-shaped booms during winding is measured (HP-10) for 5 times by using a digital display push-pull meter, respectively. And the peak torque is calculated. The result shows that the deviation between the experimental value and the theoretical value is no more than 8.83% and the average deviation is 2.813%, thus the accuracy of the theoretical model is verified. This study can be used to analyze the peak moment of the super-elastic boom in pure bending state, which is of great significance for the design of the super-elastic boom.

Key words: folding mechanism; N-shaped boom; covariant basis vector method; winding and bending; theoretical modeling

由于航天器的运载存储能有限, 对大型空间可展开结构的使用越来越多^[1], 如太阳帆^[2]、反射器天

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线等。超弹性杆因其性能优异而成为可展结构的重要组成部分,它通过释放在弯曲缠绕过程中储存的应变能实现自展开^[3-4]。在宇航空间中,超弹性杆能否顺利展开关乎空间任务的成败。目前在研究中的超弹性杆有带簧、豆芽杆、人形杆等,为了研究超弹性杆的力学性能,国内外学者已经通过解析法、试验验证和仿真法等对其进行了多方面研究。Kwok 等^[5]在考虑黏弹性的基础上推导了带簧弯曲后力矩松弛的解析表达式。Pellegrino 等^[6]研究了螺旋缠绕的带簧的稳定性;Footdale 等^[7]以带簧为臂架设计了一种自展开薄膜机构;Yasuyuki 等^[8]推导了带簧绕轮毂时的缠绕力矩和应变能的理论模型,并用实验验证了理论的准确性;Leclerc 等^[9]提出了一种人形杆的制备方法,并对人形杆在卷曲和展开时的应力集中和失效行为进行了研究;Stabile 等^[10]采用仿真法对 C 形杆的压扁缠绕进行分析,通过参数分析得到了强度较好的 C 形杆参数;叶红玲等^[11]对折叠缠绕时的带簧进行了几何参数优化。

白江波等^[12-13]采用真空袋成形和复合工艺制备了豆芽杆,并采用有限元分析方法和真空试验探究了豆芽杆在空间环境中的热行为;Chu 等^[14]提出一种豆芽杆为骨架的可展开机构,并对其动力学行为和尺寸参数的鲁棒自适应性进行了研究;Chen 等^[15]对豆芽杆的拉扁、压扁过程进行了实验和仿真分析比较;Li 等^[16]基于试验和仿真分析,对豆芽杆复合材料的缠绕过程进行分析;中国科学院的范文杰等^[17]提出了一种新型 X 形截面弹性杆,在同样的收拢高度下,它具有更高的截面惯性矩。Yang 等^[18]以展开状态的弯曲刚度和扭转刚度对 N 形杆的几何参数进行了优化,但对其缠绕过程中的力矩还未从理论上进行分析。本文对其缠绕过程进行理论分析和试验验证。N 形杆如图 1 所示,N 形杆是由 3 个横截面具有一定曲率的薄壁壳体粘贴在一起组成的。基于协变量向量法建立了单片带簧的应变能和弯矩的理论模型,然后建立了扁平段和偏离对称轴一定距离的带簧的弯矩模型,最后通过

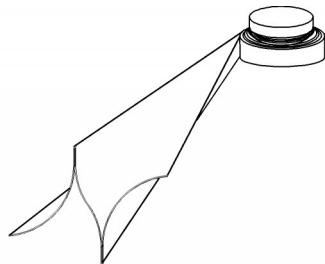


图 1 N 形杆截面

Fig.1 Cross section of N-shaped boom

拟合曲率的方法建立了 N 形杆弯矩的理论模型。研制了 4 根相同的 N 形杆样件,搭建了试验平台,分别测试了 4 根 N 形杆在缠绕时的拉力,求出出力矩,通过试验来验证 N 形杆理论模型的准确性。

1 单片带簧力矩建模

1.1 几何关系

带簧如图 2(a)所示,曲率半径为 R ,厚度为 t ,圆心角为 θ 。建立惯性笛卡尔坐标 (i_1, i_2, i_3) ,正交坐标系 (x, y, z) 和对应的标准正交基向量 (H_x, H_y, H_z) 。 x 沿平行于 i_1 的纵向, y 轴沿弧线方向, z 轴沿厚度方向指向圆心。 (H_x, H_y, H_z) 与 (i_1, i_2, i_3) 的关系如下

$$\begin{cases} H_x = i_1 \\ H_y = \cos\left(\frac{y}{R}\right)i_2 + \sin\left(\frac{y}{R}\right)i_3 \\ H_z = -\sin\left(\frac{y}{R}\right)i_2 + \cos\left(\frac{y}{R}\right)i_3 \end{cases} \quad (1)$$

壳层中任意一点在初始状态下的位置向量 X 表示为

$$X = (z - R)H_z + xi_1 \quad (2)$$

对其 x, y, z 方向求导,可得协变基向量 G_x, G_y, G_z 为

$$\begin{cases} G_x = \frac{\partial X}{\partial x} = i_1 \\ G_y = \frac{\partial X}{\partial y} = (1 - k_0 z)H_y \\ G_z = \frac{\partial X}{\partial z} = H_z \end{cases} \quad (3)$$

式中 $k_0 = 1/R$ 。

将带簧缠绕在半径为 a 的轮毂上,坐标系变化如图 2(b)所示。对应的正交基向量记为 (h_x, h_y, h_z) ,它与 (i_1, i_2, i_3) 的关系如下

$$\begin{cases} h_x = \cos\left(\frac{x}{a} - \frac{l}{2a}\right)i_1 - \sin\left(\frac{x}{a} - \frac{l}{2a}\right)i_3 \\ h_y = i_2 \\ h_z = \sin\left(\frac{x}{a} - \frac{l}{2a}\right)i_1 + \cos\left(\frac{x}{a} - \frac{l}{2a}\right)i_3 \end{cases} \quad (4)$$

带簧在纯弯曲缠绕状态下的小位移为 (u, v, w) ,在 Kirchhoff-Love 假设下,变形状态下的位置向量 x 可表示为

$$x = x_0 + \frac{z}{\left| \frac{\partial x_0}{\partial x} \times \frac{\partial x_0}{\partial y} \right|} \frac{\partial x_0}{\partial x} \times \frac{\partial x_0}{\partial y} \quad (5)$$

式中 x_0 为带簧中面的位置向量,即

$$x_0 = uh_x + (y + v)h_y + (a + w)h_z \quad (6)$$

假设 u 是关于 x 的函数; v, w 是关于 y 的函数;

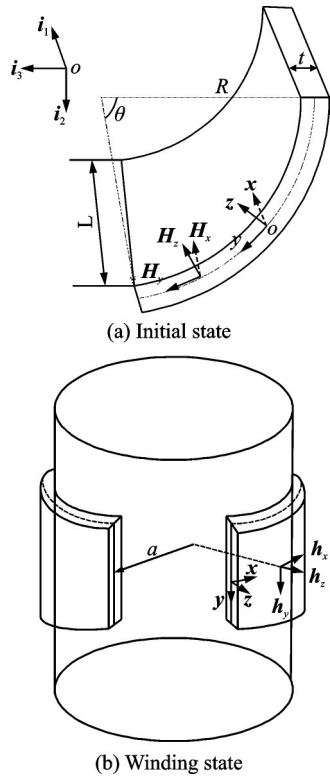


图2 初始和缠绕状态

Fig.2 Initial and winding states

带簧的厚度 t 远远小于半径 R 和长度 L , 且壳体表面上的应力是均匀分布的。

对 x 求导, 对应的协变基矢量 g_x, g_y, g_z 为

$$\begin{cases} g_x = \frac{\partial x}{\partial x} = (1 + \dot{u} + k\omega + kz)h_x - (ku)h_z \\ g_y = \frac{\partial x}{\partial y} = (1 + v' - zw'')h_y + w'h_z \\ g_z = \frac{\partial x}{\partial z} = h_z + kuh_x - w'h_y \end{cases} \quad (7)$$

式中 $k = 1/a$ 。

对称弯曲变形时只有 x, y, z 方向的轴向应变, 由前面的假设可以知道应变项中的 t/R 的二阶或者更高阶可以忽略不计, 则 x, y 轴的应变 ϵ_{xx} 和 ϵ_{yy} 可表示为

$$\begin{cases} T_x = \int_{-t/2}^{t/2} s^{xx} (1 - k_0 z) dz, T_y = \int_{-t/2}^{t/2} s^{yy} (1 - k_0 z) dz \\ M_x = \int_{-t/2}^{t/2} z s^{xx} (1 - k_0 z) dz, M_y = \int_{-t/2}^{t/2} -z s^{yy} (1 - k_0 z) dz \\ N_y = \int_{-t/2}^{t/2} z \frac{\partial s^{yy}}{\partial y} (1 - k_0 z) dz \end{cases} \quad (12)$$

将式(11)和式(12)代入壳体平衡方程式(10), 可得

$$\bar{D}_y w'''' + E_x t k^2 (\omega - \omega_0) = 0 \quad (13)$$

式中: $\bar{D}_y = \frac{E_y t^3}{12(1 - \nu_x \nu_y)} \left(1 - \frac{t^2 k_0^2}{12} \right), \omega_0 = \frac{t^2 k_0}{12}$

$$\begin{cases} \epsilon_{xx} = \frac{|g_x|}{|G_x|} - 1 \approx \dot{u} + k(\omega + z) \\ \epsilon_{yy} = \frac{|g_y|}{|G_y|} - 1 \approx v' + z(k_0 - w'') \end{cases} \quad (8)$$

1.2 平衡方程

壳体受力微元图如图3所示, 假设沿 z 方向的表面力为 p_z, p_z 表示缠绕时产生的接触应力, 且没有其他表面力。运用虚位移原理, 有

$$\int_0^L \int_{-b_1}^{b_1} \left[\int_{-t/2}^{t/2} (s^{xx} \delta e_{xx} + s^{yy} \delta e_{yy}) (1 - k_0 z) dz - p_z \delta w \right] dy - 2T_{edge} \delta v(b_1) - 2N_{edge} \delta w(b_1) - 2M_{edge} \delta w'(b_1) dx - T_{end} \delta u(l) - M_{end} l \delta_k = 0 \quad (9)$$

式中: $b_1 = 1/2R\theta_1$; s^{xx}, s^{yy} 分别表示 x, y 方向应力; $\delta e_{xx}, \delta e_{yy}, \delta w, \delta v, \delta u$ 为虚应变; $T_{edge}, N_{edge}, M_{edge}$ 分别表示在 $y = b_1$ 时, y 方向, z 方向每单位长度的外力和 x 方向每单位长度的外力矩; T_{end}, M_{end} 分别为 x 方向外力和 y 方向外力矩。

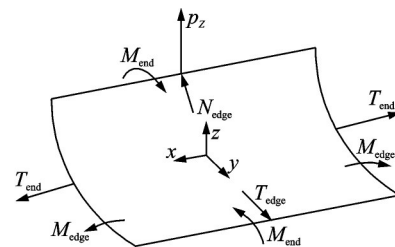


图3 壳体单位受力图

Fig.3 Diagram of shell unit force

由式(8)直接得虚应变, 即

$$\begin{cases} [\delta e_{xx} = \delta u + k\delta w + \omega + z\delta k] \\ [\delta e_{yy} = \delta v' - z\delta w''] \end{cases} \quad (10)$$

将式(10)代入式(9), 得平衡方程

$$\begin{cases} \frac{\partial T_x}{\partial x} = 0, \frac{\partial T_y}{\partial y} = 0 \\ \frac{\partial^2 M_y}{\partial y^2} + kT_x - p_z = 0 \end{cases} \quad (11)$$

纯弯曲条件下力和力矩的定义如下

$\frac{\dot{u}}{k}, E_y$ 为松弛模量, ν_x, ν_y 为泊松比。

挠度 w 可表示为双曲函数和三角函数之积的形式

$$w = \omega_0 + b_1 C_1 \cosh(\eta\xi) \cos(\eta\xi) + b_1 C_2 \sinh(\eta\xi) \sin(\eta\xi) \quad (14)$$

式中: C_1, C_2 为待定系数; ξ 为量纲为一的弧长, $\xi =$

$$\frac{y}{b_1}; \eta \text{ 为量纲为一的参数, } \eta = \sqrt[4]{\frac{E_x b_1^4 t k^2}{4 \bar{D}_y}}$$

如果没有外力, 没有边界力, 在纯弯曲条件下, 有

$$T_y(b_1) = 0, M_y(b_1) = 0, N_y(b_1) = 0 \quad (15)$$

将式(15)代入式(11), 得

$$w''(-b_1) = k_0 + v_x k, w'''(-b_1) = 0 \quad (16)$$

再将式(16)代入式(14), 解得

$$w = \frac{t^2 k_0}{12} - \frac{\dot{u}}{k} + \frac{b_1^2 (k_0 + v_x k)}{3} g(\xi) \quad (17)$$

式中

$$g(\xi) = \frac{3(\sinh \eta \cos \eta + \cosh \eta \sin \eta)}{\eta^2 (\sinh(2\eta) + \sin(2\eta))} \sinh(\eta \xi) \sin(\eta \xi) - \frac{3(\cosh \eta \sin \eta - \sinh \eta \cos \eta)}{\eta^2 (\sinh(2\eta) + \sin(2\eta))} \cosh(\eta \xi) \cos(\eta \xi)$$

1.3 应变能和力矩

单带簧中每单位长度存储的应变能 π 表示为

$$\pi = \int_{-b_1}^{b_1} \int_{-t/2}^{t/2} \frac{1}{2} (\sigma^{xx} \epsilon_{xx} + \sigma^{yy} \epsilon_{yy}) (1 - k_0 z) dz dy \quad (18)$$

带簧片先经历了由凸到平的变形, 后经历了弯曲缠绕过程, 即

$$\pi = \pi_x + \pi_y \quad (19)$$

式中: π_x 为由凸变平的应变能, π_y 为缠绕产生的应变能, 即

$$\pi_x = \frac{k}{2} \int_0^{b_1} M_x dy, \pi_y = -\frac{k_0}{2} \int_0^{b_1} M_y dy \quad (20)$$

将式(12)和式(20)代入式(19), 得

$$\pi = b_1 \bar{D}_x (1 - v_x v_y) k^2 + b_1 \bar{D}_y (k_0 + v_x k)^2 (1 - A_1) \quad (21)$$

式中: $\bar{D}_x = \frac{E_x t^3}{12(1 - v_x v_y)} \left(1 - \frac{t^2 k_0^2}{12} \right)$ 表示为弯曲刚度, E_x 为松弛模量, $A_1 = \frac{\cosh(2\eta) - \cos(2\eta)}{\eta(\sinh(2\eta) + \sin(2\eta))}$ 。

由最小势能原理得弯矩公式

由最小势能原理得弯矩公式

$$M = \frac{d\pi}{dk} \quad (22)$$

将式(21)代入式(22)可得

$$M = 2b_1 \bar{D}_x (1 - v_x v_y) k + 2b_1 v_x \bar{D}_y (v_x k) (1 - A_1) - b_1 \bar{D}_y (k_0 + v_x k)^2 \frac{A_4 - A_1}{4k} \quad (23)$$

式中 $A_4 = \frac{4\sinh(2\eta)\sin(2\eta)}{(\sinh(2\eta) + \sin(2\eta))^2}$ 。

1.4 扁平段弯矩模型

扁平段如图 4 所示, 由前述分析可知, 扁平段的应变能可表示为

$$M = 2b_1 \bar{D}_x (1 - v_x v_y) k + 2b_1 v_x \bar{D}_y (k_0 + v_x k) (1 - A_1) - b_1 \bar{D}_y (k_0 + v_x k)^2 \frac{A_4 - A_1}{2k} \quad (33)$$

$$\pi_l = \pi_{xl} + \pi_{yl} \quad (24)$$

式中: π_{xl} 为扁平段由凸变平的应变能, π_{yl} 为缠绕产生的应变能。

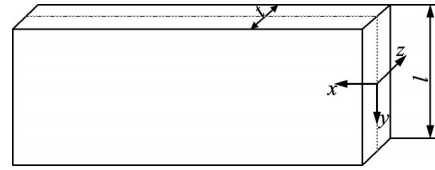


图 4 扁平段

Fig.4 Flat segment

扁平段的初始曲率为 0, 在缠绕时它只经历了曲率为 k 的缠绕过程, 即

$$\pi_l = l \bar{D}_x (1 - v_x v_y) k^2 + l \bar{D}_y (v_x k)^2 (1 - A_1) \quad (25)$$

由最小势能原理可得

$$M_l = 2l \bar{D}_x (1 - v_x v_y) k + 2l v_x \bar{D}_y (v_x k) (1 - A_1) - l \bar{D}_y (v_x k)^2 \frac{A_4 - A_1}{4k} \quad (26)$$

2 离对称轴距离为 S 的带簧弯矩建模

2.1 几何关系和挠度

带簧如图 5(a) 所示, 与第 1.1 节图 2(a) 相同, 建立相同的惯性笛卡尔坐标 (i_1, i_2, i_3) 、正交坐标系 (x, y, z) 和对应的标准正交基向量 (H_x, H_y, H_z) 。N 形杆横截面如图 5(b) 所示, 其外侧带簧离对称轴的距离为 s , 在缠绕后其中面的位置向量 x 表示为

$$x = u h_x + (y + u) h_y + (a + w + s) h_z \quad (27)$$

求得协变基矢量为

$$\begin{cases} g_x = (1 + \dot{u} + k w + k z + k s) h_x - k u h_z \\ g_y = (1 + v' - z w'') h_y + w' h_z \\ g_z = h_z + k u h_x - w' h_y \end{cases} \quad (28)$$

对应的 x, y 方向的应变为

$$\begin{cases} \epsilon_{xx} \approx \dot{u} + k(w + z + s) \\ \epsilon_{yy} \approx v' + z(k_0 - w'') \end{cases} \quad (29)$$

由于边界条件相同, 建立的平衡方程与式(11)相同, 经计算可得挠度 w 为

$$w = \frac{t^2 k_0}{12} - \frac{\alpha_{x0}}{k} - s + \frac{b_1^2 (k_0 + v_x k)}{3} g(\xi) \quad (30)$$

2.2 应变能和力矩

由 1.3 节分析可知, 带簧的应变能可表示为

$$\pi_2 = \pi_{x2} + \pi_{y2} \quad (31)$$

经计算, 可得应变能和弯矩为

$$\pi = b_1 \bar{D}_x (1 - v_x v_y) k^2 + b_1 \bar{D}_y (k_0 + v_x k)^2 (1 - A_1) \quad (32)$$

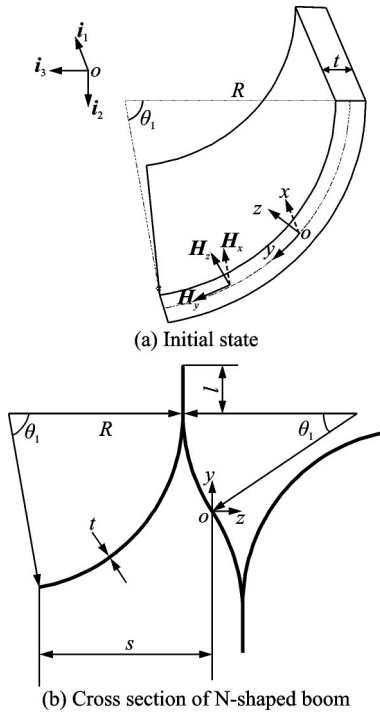


图5 N形杆截面和带簧初始状态

Fig.5 Initial state of tape spring and cross section of N-shaped boom

式中： $\bar{D}_x = \frac{E_x t^3}{12(1-\nu_x \nu_y)} \left(1 - \frac{t^2 k_0^2}{12}\right)$, $\bar{D}_y = \frac{E_y t^3}{12(1-\nu_x \nu_y)} \left(1 - \frac{t^2 k_0^2}{12}\right)$ 表示弯曲刚度, E_x 、 E_y 为松弛模量, $b_1 = 1/2R\theta_1$ 表示截面宽度的一半, k 为曲率。

采用协变基向量法对偏离对称轴距离为0和S的带簧的理论推导中,发现两者的弯矩和应变能的理论模型相同,但在挠度上有差别,相差的大小为S。下面将推导N形杆的弯矩解析解。

3 N形杆理论建模

3.1 几何分析转化

N形杆初始状态如图6(a)所示。在初始状态下,左右反对称分布,对称中心左右两部分都是由一端圆心角为 θ_1 、半径为R的带簧,一段粘合的线段l,以及一段圆心角为 θ_2 、半径为R的带簧组成,每一片的厚度均为t。

N形杆缠绕后的曲率如图6(b)所示,中心曲率为 $k = 1/a_0$,内边曲率 $k_{in} = 1/(a - 3t/2)$,外边曲率为 $k_{out} = 1/(a + 3t/2)$ 。以N形杆曲率半径 a_0 处为分界线,将其分为内外两组。

内组为杆1和杆2对称分布,其中心曲率为

$$\begin{aligned} \pi_{in1} &= b_1 \bar{D}_x (1 - \nu_x \nu_y) k_A^2 + b_1 \bar{D}_y (k_0 + \nu_x k_A)^2 (1 - A_1(k_A, b_1, k_0)) + \\ &\quad l \bar{D}_x (1 - \nu_x \nu_y) k_A^2 + l \bar{D}_y (\nu_x k_1)^2 (1 - A_1(k_A, l, 0)) \\ \pi_{out2} &= b_2 \bar{D}_x (1 - \nu_x \nu_y) k_A^2 + b_2 \bar{D}_y (k_0 + \nu_x k_A)^2 (1 - A_1(k_A, b_1, -k_0)) + \end{aligned} \quad (38)$$

$k_A = 1/(a_0 - t/2)$, 内边曲率为 $k_1 = 1/(a_0 - 3t/2)$, 外边曲率为 $k_2 = 1/a_0$ 。外组中杆3和杆4对称分布,其中心曲率为 $k_B = 1/(a_0 + t/2)$,内边曲率为 $k_3 = 1/a_0$,外边曲率为 $k_4 = 1/(a_0 + 3t/2)$ 。

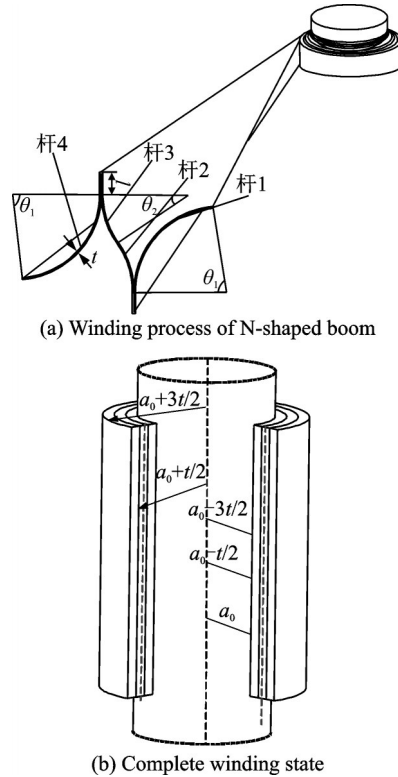


图6 N形杆缠绕过程和完全缠绕的状态

Fig.6 Winding process and state of complete winding of N-shaped boom

3.2 应变能的计算

缠绕一圈后,N形杆的应变能分配可表示为

$$\pi_N = \frac{k}{k_{in}} \pi_{in0} + \frac{k}{k_{out}} \pi_{out0} \quad (34)$$

式中: π_{in0} 为内组两片杆的应变能, π_{out0} 为外组两片杆应变能,即

$$\pi_{in0} = \frac{k_A}{k_1} \pi_{in1} + \frac{k_A}{k_2} \pi_{out1} \quad (35)$$

$$\pi_{out0} = \frac{k_B}{k_3} \pi_{in2} + \frac{k_B}{k_4} \pi_{out2} \quad (36)$$

将式(35~36)代入式(34),得

$$\pi_N = \frac{k}{k_{in}} \left(\frac{k_A}{k_1} \pi_{in1} + \frac{k_A}{k_2} \pi_{out1} \right) + \frac{k}{k_{out}} \left(\frac{k_B}{k_3} \pi_{in2} + \frac{k_B}{k_4} \pi_{out2} \right) \quad (37)$$

式中, π_{in1} 和 π_{out2} 是由一段圆心角为 θ_1 、半径为R的带簧($b_2 = 1/2R\theta_2$, $b_1 = 1/2R\theta_1$)加一段长为l的扁平段的应变能组成,但它俩初始曲率方向不同,即

$$l\bar{D}_x(1 - \nu_x \nu_y)k_A^2 + l\bar{D}_y(\nu_x k_A)^2(1 - A_1(k_A, l, 0)) \tag{39}$$

式中 $A_1(k_A, b_1, k_0)$ 表示在长度为 b_1 时, 曲率从 k_0 到 k_A 变化后 A_1 的值。

π_{out1} 和 π_{in2} 都可以看作由厚度为 t 、圆心角为 θ_2 、半径为 R 的带簧加一段长为 l 的扁平段应变能组成

$$\pi_{out1} = b_2 \bar{D}_x(1 - \nu_x \nu_y)k_B^2 + b_2 \bar{D}_y(k_0 + \nu_x k_B)^2(1 - A_1(k_B, b_2, k_0)) + l\bar{D}_x(1 - \nu_x \nu_y)k_B^2 + l\bar{D}_y(\nu_x k_B)^2(1 - A_1(k_B, l, 0)) \tag{40}$$

$$\pi_{in2} = b_1 \bar{D}_x(1 - \nu_x \nu_y)k_B^2 + b_1 \bar{D}_y(k_0 + \nu_x k_B)^2(1 - A_1(k_B, b_1, -k_0)) + l\bar{D}_x(1 - \nu_x \nu_y)k_B^2 + l\bar{D}_y(\nu_x k_B)^2(1 - A_1(k_B, l, 0)) \tag{41}$$

综上, 将式(38~41)代入式(37), 得到 N 形杆纯弯曲时应变能为

$$\pi_N = \frac{(2 - 3tk)^2}{4 - 2tk} W_1 + \frac{8 - 6t^2 k^2}{4 - 2tk} W_2 + 2W_3 + \frac{(2 + 3tk)^2}{4 + 2tk} W_4 \tag{42}$$

式中: $W_1 = b_2 \bar{D}_x s k_1^2 + b_2 \bar{D}_y P_1^2(1 - B_{12}) + l\bar{D}_x s k_1^2 + l\bar{D}_y(\nu_x k_1)^2(1 - B_{13})$, $W_3 = lD_x s k^2 + lD_y(\nu_x k)^2(1 - B_{03})$, $P = k_0 + \nu_x k$, $W_2 = b_1 D_x s k^2 + b_1 D_y P^2(1 - B_{01}) + b_1 D_x s k^2 + b_1 D_y \bar{P}^2(1 - B_{01})$, $P_1 = k_0 + \nu_x k_1$, $\bar{P} = -k_0 + \nu_x k$, $\bar{P}_4 = -k_0 + \nu_x k_4$, $s = 1 - \nu_x \nu_y$, $W_4 = b_2 \bar{D}_x s k_4^2 + b_2 \bar{D}_y \bar{P}_4^2(1 - B_{42}) + l\bar{D}_x s k_4^2 + l\bar{D}_y(\nu_x k_4)^2(1 - B_{43})$, $D_x =$

$$\frac{E_x t^3}{12(1 - \nu_x \nu_y)}, \quad \bar{D}_x = \frac{E_x t^3}{12(1 - \nu_x \nu_y)} \left(1 - \frac{t^2 k_0^2}{12} \right), \quad \eta_{01} = \sqrt[4]{\frac{E_x b_1^4 t k^2}{8D_y}}, \quad B_{01} = \frac{\cosh(2\eta_{01}) - \cos(2\eta_{01})}{\eta_{01}(\sinh(2\eta_{01}) + \sin(2\eta_{01}))}, \quad B_{03} = \frac{\cosh(2\eta_{03}) - \cos(2\eta_{03})}{\eta_{03}(\sinh(2\eta_{03}) + \sin(2\eta_{03}))},$$

$$B_{12} = \frac{\cosh(2\eta_{12}) - \cos(2\eta_{12})}{\eta_{12}(\sinh(2\eta_{12}) + \sin(2\eta_{12}))}, \quad B_{13} = \frac{\cosh(2\eta_{13}) - \cos(2\eta_{13})}{\eta_{13}(\sinh(2\eta_{13}) + \sin(2\eta_{13}))}, \quad B_{42} = \frac{\cosh(2\eta_{42}) - \cos(2\eta_{42})}{\eta_{42}(\sinh(2\eta_{42}) + \sin(2\eta_{42}))},$$

$$B_{43} = \frac{\cosh(2\eta_{43}) - \cos(2\eta_{43})}{\eta_{43}(\sinh(2\eta_{43}) + \sin(2\eta_{43}))}, \quad \eta_{03} = \sqrt[4]{\frac{E_x l^4 t k^2}{D_y}}, \quad \eta_{12} = \sqrt[4]{\frac{E_x b_2^4 t k_1^2}{4\bar{D}_y}}, \quad \eta_{42} = \sqrt[4]{\frac{E_x b_2^4 t k_4^2}{4\bar{D}_y}}, \eta_{43} = \sqrt[4]{\frac{E_x l^4 t k_4^2}{4D_y}}, \eta_{13} = \sqrt[4]{\frac{E_x l^4 t k_1^2}{4D_y}}.$$

3.3 弯矩的求解

基于最小势能原理, 可以推导出弯矩为

$$M_N = \frac{d\pi_N}{dk} \tag{43}$$

将式(42)代入式(43), 得

$$M_N = \left(\frac{k \cdot k_A}{k_1^2} \right)' \pi_{in1} + \frac{k \cdot k_A}{k_1^2} \pi_{in1}' + \left(\frac{k \cdot k_A}{k_1 \cdot k_2} \right)' \pi_{out1} + \frac{k \cdot k_A}{k_1 \cdot k_2} \pi_{out1}' + \left(\frac{k \cdot k_B}{k_3 \cdot k_4} \right)' \pi_{in2} + \frac{k \cdot k_B}{k_3 \cdot k_4} \pi_{in2}' + \left(\frac{k \cdot k_B}{k_4^2} \right)' \pi_{out2} + \frac{k \cdot k_B}{k_4^2} \pi_{out2}' \tag{44}$$

将式(38~41)代入式(44), 得到 N 形杆纯弯曲时弯矩为

$$M_N = \frac{8 - 4.5(2 - tk)^2}{(2 - tk)^2} tQ_1 + \frac{2}{2 - tk} Q_2 - \frac{32t^2 k}{(2 - tk)^2(2 + tk)^2} Q_3 + \frac{8 - 6t^2 k^2}{4 - t^2 k^2} Q_4 + \frac{4.5(2 + tk)^2 - 8}{(2 + tk)^2} tQ_5 + \frac{2}{2 + tk} Q_6 \tag{45}$$

式中: $Q_1 = b_2 \bar{D}_x s k_1^2 + b_2 \bar{D}_y P_1^2(1 - B_{12}) + l\bar{D}_x s k_1^2 + l\bar{D}_y(\nu_x k_1)^2(1 - B_{13})$, $N_1 = \frac{8 - 4.5(2 - tk)^2}{(2 - tk)^2}$, $N_3 =$

$$\frac{32t^2 k}{(2 - tk)^2(2 + tk)^2}, \quad Q_2 = 2b_2 \bar{D}_x s k_1 + 2b_2 \nu_x \bar{D}_y P_1(1 - B_{12}) - b_2 \bar{D}_y P_1^2 \frac{C_{12} - B_{12}}{2k_1} + 2l\bar{D}_x s k_1 + 2l\nu_x \bar{D}_y(\nu_x k_1)(1 - B_{13}) - b_2 \bar{D}_y(\nu_x k_1)^2 \frac{C_{13} - B_{13}}{2k_1},$$

$$N_2 = \frac{2}{2 - tk}, \quad Q_3 = b_1 D_x s k^2 + b_1 D_y P^2(1 - B_{01}) + b_1 D_x s k^2 + b_1 D_y \bar{P}^2(1 - B_{01}) + 2[lD_x s k^2 + lD_y(\nu_x k)^2(1 - B_{03})],$$

$$N_5 = \frac{4.5(2 + tk)^2 - 8}{(2 + tk)^2}, \quad Q_4 = Q_{41} - Q_{42} + Q_{43}, \quad Q_{41} = 2b_1 D_x s k + 2b_1 \nu_x D_y P(1 - B_{01}),$$

$$Q_{43} = 2[2lD_x s k + 2l\nu_x D_y(\nu_x k)(1 - B_{03}) - lD_y(\nu_x k)^2 \frac{C_{03} - B_{03}}{2k}], \quad Q_{42} =$$

$$b_1 D_y P_1^2 \frac{C_{01} - B_{01}}{2k} + 2b_1 D_x s k + 2b_1 \nu_x D_y \bar{P}(1 - B_{01}) - b_1 D_y \bar{P}_1^2 \frac{C_{01} - B_{01}}{2k}, \quad N_4 = \frac{8 - 6t^2 k^2}{4 - t^2 k^2}, \quad C_{12} =$$

$$\frac{4\sinh(2\eta_{12})\sin(2\eta_{12})}{(\sinh(2\eta_{12})+\sin(2\eta_{12}))^2}, \quad Q_5 = b_2\bar{D}_x sk_4^2 + b_2\bar{D}_y \bar{P}_4^2(1-B_{42}) + \bar{D}_x sk_4^2 + \bar{D}_y (v_x k_4)^2(1-B_{43}), \quad C_{43} =$$

$$\frac{4\sinh(2\eta_{43})\sin(2\eta_{43})}{(\sinh(2\eta_{43})+\sin(2\eta_{43}))^2}, \quad C_{01} = \frac{4\sinh(2\eta_{01})\sin(2\eta_{01})}{(\sinh(2\eta_{01})+\sin(2\eta_{01}))^2}, \quad Q_6 = 2b_2\bar{D}_x sk_4 + 2b_2v_x\bar{D}_y \bar{P}_4(1-B_{42}) -$$

$$l\bar{D}_y (v_x k_4)^2 \frac{C_{43}-B_{43}}{2k_4} - b_2\bar{D}_y \bar{P}_4^2 \frac{C_{42}-B_{42}}{2k_4} + 2l\bar{D}_x sk_4 + 2lv_x\bar{D}_y (v_x k_4)(1-B_{43}), \quad N_6 = \frac{2}{2+tk}, \quad C_{03} =$$

$$\frac{4\sinh(2\eta_{03})\sin(2\eta_{03})}{(\sinh(2\eta_{03})+\sin(2\eta_{03}))^2}, \quad C_{13} = \frac{4\sinh(2\eta_{13})\sin(2\eta_{13})}{(\sinh(2\eta_{13})+\sin(2\eta_{13}))^2}, \quad C_{42} = \frac{4\sinh(2\eta_{42})\sin(2\eta_{42})}{(\sinh(2\eta_{42})+\sin(2\eta_{42}))^2}$$

3.4 理论曲线

N形杆材料参数如表1所示,其外侧圆弧角 $\theta_1=1.45$ rad,内侧圆弧角 $\theta_2=0.45$ rad,直线段长度 $l=11$ mm,厚度 $t=0.14$ mm,初始曲率半径为 $R=20$ mm。将参数代入式(33),得到力矩-曲率曲线如图7所示。在开始时,力矩随曲率迅速上升到峰值 0.2425 N·m,之后又迅速下降,趋于稳定。

表1 N形杆材料参数

Table 1 Material parameters of N-shaped boom

材料属性	值
泊松比	0.3
弹性模量 E_x /GPa	33.24
弹性模量 E_y /GPa	32.63
密度 ρ /(kg·m ⁻³)	1 600

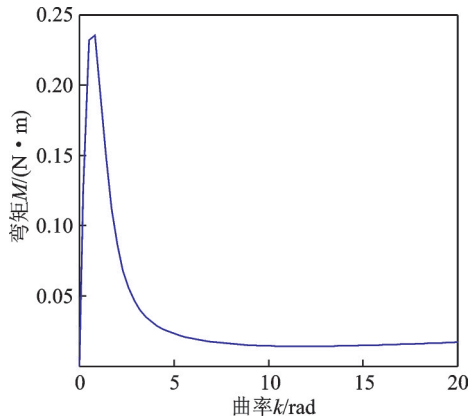


图7 N形杆理论曲线

Fig.7 Theoretical curve of N-shaped boom

峰值力矩能够表征N形杆在展开状态下抵抗外力的能力,代表着整个机构的刚度大小,为此,搭建试验平台对其进行测量。

4 N形杆缠绕试验

为了进行N形杆缠绕试验,研制了4根相同的N形杆样件如图8(a)所示,搭建试验平台对缠绕过程中的力矩进行测试,试验平台如图8(b)所示。将N形杆一端固定于轮毂,利用推拉式力计带动连接在轮毂上的臂杆旋转,进而带动N形杆实现缠绕。推拉式力计的型号为(HP-10),量程为10 N,灵敏度

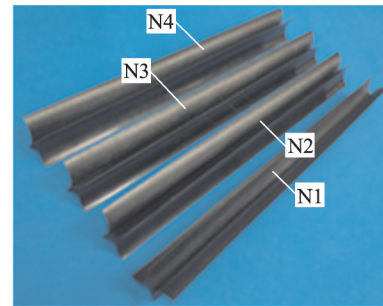
为0.001 N,其另一端连接数据采集系统,可输出测试过程中的拉力。

推拉力计测量的拉力乘以力臂长度,可以得到N形杆缠绕时的力矩 M ,即

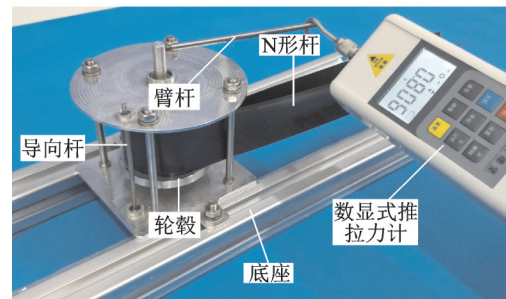
$$M = F \cdot L \quad (46)$$

式中: M 和 F 分别为力矩和拉力, L 为力臂长度。

四根N形杆样件分别用N1、N2、N3和N4表示,对每根N形杆分别进行了5次测试,得到试验的峰值力矩、峰值力如表2所示。



(a) Four N-shaped booms



(b) Experimental device

图8 N形杆和弯曲试验装置

Fig.8 N-shaped booms and experimental device

为了验证理论模型的准确性,用偏差和偏差标准差来表示理论值和试验值误差的大小,即

$$RE_i = \frac{M_e^i - M_r}{M_e^i} \times 100\% \quad (47)$$

$$\Delta = \sqrt{\frac{1}{n} \sum_{i=1}^n (RE_i - \bar{\delta})^2} \quad (48)$$

式中: M_e^i 、 M_r 分别为试验值、理论值, i 为第 i 个测试数据, n 为测试总数, $\bar{\delta}$ 为偏差均值。

将表2中测得的峰值力矩代入式(47)中对应的试验值 M_e^i ,得到N形杆缠绕过程中,试验值与理

论值的偏差值不大于8.83%,偏差平均值为2.813%,偏差标准差为4.97%,验证了理论模型的准确性。但其中仍存在误差,误差来源主要有以下几方面:N形杆与轮毂之间、N形杆与导向杆之间存在摩擦;推拉力计与力臂之间存在小幅度滑动;在缠绕过程中,推拉力计的方向与力臂的方向不是完全垂直的;试验件N形杆的尺寸存在偏差。

表2 4根N形杆的峰值力和峰值力矩

Table 2 Peak forces and peak torques of four N-shaped booms

样件代号	序号	峰值力/N	峰值力矩/ (N·m)	RE/%
N1	1	2.390	0.265	8.49
	2	2.341	0.257	5.64
	3	2.258	0.251	3.39
	4	2.336	0.256	5.27
	5	2.387	0.262	7.44
N2	1	2.099	0.230	-5.43
	2	2.236	0.246	1.42
	3	2.022	0.224	-8.26
	4	2.144	0.236	-2.75
	5	2.360	0.259	6.37
N3	1	2.289	0.263	7.79
	2	2.243	0.258	6.01
	3	2.168	0.249	2.62
	4	1.996	0.230	-5.43
	5	2.278	0.262	7.44
N4	1	2.122	0.244	0.61
	2	2.310	0.266	8.83
	3	2.075	0.239	-1.46
	4	2.174	0.250	3.0
	5	2.229	0.256	5.27

5 结 论

为防止N形杆在缠绕时的失稳,通过理论建模得到了N形杆在缠绕时的峰值力矩。通过试验验证并分析参数,得到了以下结论:

(1)基于协变量向量法建立了单带簧的应变能理论模型,采用最小势能原理推导了弯矩的表达式,然后采用相同方法对扁平段和偏离对称轴一定距离的带簧建立弯矩模型,最后拟合曲率建立了N形杆弯矩的理论模型。

(2)搭建试验平台,研制了4根N形杆样件,分别对每个N形杆弯曲时的缠绕力矩进行5次测量,并将其与理论值对比,得到偏差值不大于8.83%、偏差平均值为2.813%、偏差标准差为4.97%。

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